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Rate Effects on Instabilities in Slightly Ionized Plasmas

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PREFACE

The work described in this report was performed by the Propulsion Division of the Jet Propulsion Laboratory.

CONTENTS

Abstract

Presented are results of numerical computer calculations of dispersive wave growth rates in slightly ionized plasmas. The effects of employing a plasma model with detailed consideration of rate and radiative processes involving the electron quantum states are studied. Acoustic and electrothermal waves in unseeded argon are found to be frozen with the latter damped. Rate-thermal modes can exhibit large growth. Mode mixing and switching in both seeded and unseeded argon are demonstrated.

Introduction

A slightly ionized plasma is used as a working fluid in many magnetofluid-dynamic devices. In particular, there has been considerable interest in the past decade in gaseous MFD power generators. Due to marked effects of instabilities on the characteristics and performance of a generator (Gilpin and Zukoski, 1969; Solbes, 1970; Dykhne, 1971; Lengyel, 1971), the question of their existence has been given much attention (Nelson and Haines, 1969; Fishman, 1970; Hiramoto, 1970; Hougen and McCune, 1971; Nemchinskii, 1972). Of more recent concern have been instabilities in a MFD laser (Oliver et al, 1972). The theoretical models that have been used to describe the plasma have either assumed that electronic excited state populations can be given by an equilibrium (Boltzmann) distribution or by use of the quasi-stationary rate approximation. The latter is discussed in Biberman, Yakubov and Vorobev (1971); it results in a single rate equation for the production of free electrons. A more general approach has been developed by the author (Harstad, 1972, hereafter referred to as I)*. A set of rate equations describing the inelastic and radiative processes governing transitions between states is utilized. It is shown in I that this more complete description is necessary to accurately describe linear dispersive wave phenomena in the plasma. Presented in this paper are results of computer calculations of linear instability growth rates using the equations of I. The preliminary results given in this reference are thus extended. Important to this extension are computations for an unseeded plasma.

The dispersion relation results from assuming time dependent small perturbations to steady state conditions. The perturbation δV of any dependent variable V is of the form $\delta V = \tilde{V} \exp [i (\vec{\kappa} \cdot \vec{x} - \omega t)]$ where $\vec{x}$ is the position coordinate

*Numerous typographical errors occur in the reference; a corrected version is available from the author.

vector, t the time, $\vec{\varkappa}$ the real wave number, and the amplitude $\tilde{V}$ and frequency ω are complex. The wavelength is taken much shorter than the plasma container dimension or any derivative length. The dispersion function $\omega(\vec{\varkappa})$ for the various modes is proportional to eigenvalues of a complex matrix whose elements are a function of the wave number, plasma properties, electromagnetic fields, etc. (I). Note that the imaginary part of ω is positive for wave growth.

For the calculations presented here, the plasma device is in the generator mode with a load parameter of 0.75 and no axial Hall current. A characteristic device dimension of 0.1 m is assumed (this is employed in a radiative escape factor). Spherical coordinates are used for the wave vector: $\vec{\varkappa} = (\varkappa, \zeta, \theta)$. Polar angle θ is that between $\vec{\varkappa}$ and the magnetic field (directed along the z-axis). Azmuthal angle ζ is defined as that from the y-axis (negative current direction), taken in the direction of the x-axis (velocity direction), to the projection of $\vec{\varkappa}$ on the x-y plane. For most of the calculations the wavelength is 1 cm. Argon with and without seed material is studied.

Results for Unseeded Argon

For seeded gases at high (near atmospheric) pressure, it is expected that the steady unperturbed excited state populations of the ionizing material are close to the equilibrium Boltzmann distribution (Kerrebrock, 1965). At low pressures or for unseeded noble gases where ground-resonance level energy gaps are very large, inelastic collisional rates are much smaller and this is not true. For those cases considered with a plasma formed from pure argon, the steady populations were computed using the quasi-stationary rate approximation. It can be expected that this is a valid procedure for the steady flow (Biberman, Yakubov and Vorobev, 1971), although it is not for the dispersive waves. The plasma was considered to be either ionizing or recombining with an electron

number density differing from the equilibrium value by a factor of roughly five. These are conditions that may be appropriate to the front (nearer the entrance) and back (nearer the exit) portions of a device respectively. The unperturbed populations can be far from equilibrium. For example, they frequently feature an inversion (up to a factor 1.6) between the 3d-5s configuration with, at times, a weaker inversion between 4d-6s and also 4f-6p for one case (Table 1, Case 8). Due to the large energy gap between the argon ground and resonance levels, populations of excited levels are relatively small.

A characteristic interaction time can be defined as $t_c = \rho/(\sigma B^2)$ where ρ is the total mass density, σ the electrical conductivity, and B the magnetic field strength. The interaction length, $L_c = ut_c$ using velocity u. A linear growth factor is defined as $G \equiv \exp(\omega_I t_c)$ where $\omega \equiv \omega_R + i\omega_I$. This factor is indicative of the amount of growth (by linear theory) that a disturbance will experience during its residence in a device with interaction parameter of order unity.

Previous experience with seeded generators (Nemchinskii, 1972; Solbes, 1970; I) indicates maximum growth of instabilities occurs in the plane $\theta = \pi/2$; present calculations support this conclusion. In all cases, close correspondence to the frozen rate limits was found for both acoustic and electrothermal modes, the latter being strongly damped. The label "thermal" is applied to the mode with the largest mass density perturbation (see Hougen and McCune, 1971; I). Strictly applied this label is only appropriate if the density perturbation is the significant part of the mode's eigenvector and if it is relatively small for other non-acoustic mode eigenvectors. However, it is applied here to all data points even if this requirement is not met, i.e., even if mode mixing occurs. Significant growth was found in some cases for the lowest order (smallest eigenvalue magnitude) rate mode associated with the rate equation set. This mode tends to mix with the thermal mode in the plane $\theta = \pi/2$ and both mix with the vorticity modes (I) for other values of the angle θ.

Table 1 gives the properties of a dozen different cases. For these cases the atom temperature equals $1000^{o}K$. Correspondingly, Table 2 gives the maximum growth factors at $\theta = \pi/2$ and a wavelength of 1 cm for acoustic and thermal-rate modes and the angles ζ (with given increments) at which they occur. At their collective maximum growth point, mixing of the thermal and rate modes is slight and the mode type is also given. At the lower pressures, only the acoustic mode gives significant growth. The acoustic growth rates vary weakly with pressure (see also I). Figure 1 shows the detailed behavior of the normalized acoustic wave growth vs angle ζ for Case 3. In Fig. 2 for this same case a plot of the acoustic wave growth slightly away from its maximum point is given as function of wave number magnitude. In contrast to the results reported in I for potassium seeded argon where the growth drops rather sharply with increasing wave number, it is seen that the growth in Fig. 2 increases slowly. (This behavior was duplicated in calculations of a device operating in an accelerator mode). Although there were occasional indications at certain data points of fairly strong mixing between acoustic and other modes, the acoustic modes tended to remain distinct. The behavior of the thermal-rate modes of Case 3 is presented in Figs. 2-4. The modes are not distinct but switch during mixing from one type to another, Fig. 3. The pronounced mixing or change of mode type occurs in a small interval of azmuthal angle of order 0.1 radian. In Fig. 4 the velocity u changes with B to maintain a constant dissipation. It is to be noted that the rate mode type points and thermal mode type points closely follow the respective behavior of the electrothermal and thermal modes as given in I. However, unlike the electrothermal mode, rate mode type growth can occur for a null magnetic field. The eigenvalues of the thermal-rate-vorticity modes of Case 3 as functions of angle θ are presented in Fig. 5. They are labeled according to their behavior at $\theta = \pi/2$. For small θ, the mixing is such that the labels are essentially

meaningless. At $\theta = \pi/2$, the null eigenvalue vorticity modes form a pair (as do the acoustic modes). As θ decreases, this changes until at small θ, the "rate" and "second vorticity" modes are paired.

Results for Seeded Argon

In cases where the argon is seeded with a metal, the argon atoms are assumed to be all in the ground state and the seed atoms in their unperturbed condition to have an equilibrium Boltzmann distribution of excited states at the electron temperature. Results using a potassium seed are given in I and need not be repeated here. A major difference of these results as compared to the unseeded cases is that all the rate modes are damped and the electrothermal mode exhibits growth and is the dominant mode. Also, for the seeded plasma, the thermal and vorticity modes tend to remain somewhat distinct. The like behavior of the thermal mode type points for seeded and unseeded argon and the electrothermal mode for seeded argon compared to the rate mode type points for unseeded argon, along with the fact that the electrothermal and thermal modes have null growth points at the same values of ζ (see Figs. 5 and 9 in I), seems to indicate a possibility of electrothermal-thermal mode mixing for the seeded cases. This idea was verified upon further numerical calculation, see Fig. 6. The mode mixing was apparent in the eigenvectors. The change of mode type occurs over a very small angle, ζ varying on the order of 10^{-2} radians in the mixing region. This is the reason that the mode type switching was overlooked in I. The fact that the electrothermal mode in seeded plasmas is non-distinct has more academic, rather than practical, significance. It does illustrate an effect that simpler plasma models do not show.

Some additional calculations for potassium seeded argon at atmospheric pressure were made to illustrate the expected damping effect on the electrothermal waves as full ionization of the seed is approached. For these

particular calculations, the seed ratio is 10^{-3}, the Hall parameter (from Ohm's law) magnitude is 6.0, the wavelength is 1 cm, $\theta = \pi/2$, $\zeta = 0.74\pi$, and T = 1500°K. The growth rate and degree of ionization of the seed are plotted against electron temperature in Fig. 7. Although the frozen and equilibrium limits must coalesce as full ionization is approached, they remain apart for the conditions considered. The figure shows that the wave growth remains near the equilibrium limit and that a high temperature ($T_e > 6000°K$) is needed for damping. The damping is thus not due to any "freezing" phenomenon but due to the ultimate decrease in seed atom quantum state populations with electron temperature increase.

In unseeded argon the electrothermal mode is nearly frozen and strongly damped; in potassium seeded argon it is frequently close to the equilibrium limit and dominant. It was decided to study argon with magnesium seed, the ionization potential of magnesium being intermediate to that of argon and potassium. Calculations were made for atmospheric pressure and the seed ratio was varied between extremes of 10^{-3} and 0.05. Detailed results will not be given, but generally speaking, the behavior of electrothermal waves was similar to that in potassium seeded argon. Electrothermal growth was large with large Hall parameter magnitude although the deviation from the equilibrium limit was more than potassium seed gives. The critical magnitude of the Hall parameter (below which damping occurs) was about 2.0 for a seed ratio of 10^{-3} and $T_e = 5000°K$.

From all the calculations made, it seems that the populations of the electron quantum states of the ionizing element is more important than its ionization potential in determining the electrothermal growth rate.

Summary and Conclusions

With the possible exception of the acoustic modes, mode mixing and type switching are common phenomena for dispersive waves in slightly ionized plasmas.

The mixing is least at the plane $\theta = \pi/2$. For seeded plasmas, it has limited practical significance in that it effects the electrothermal "mode" only near its apparent null growth points.

For unseeded noble gases, the acoustic modes can be considered frozen for pressures on the order of atmospheric or less. The growth rate increases with wave number. This means that viscous effects should be included in the model if accurate prediction of their behavior is desired.

Damping of electrothermal waves at large magnitudes of the Hall parameter requires that quantum state populations be small. This occurs if the ground level-resonance level energy gap is sufficiently large or, in the case of a seeded plasma, if full ionization of the seed is approached. This means damping can occur in both seeded and unseeded noble gases for a large electron temperature - needed either for full seed ionization or sufficient electron density production respectively. Unfortunately, this also means radiative energy losses may tend to be high in nonequilibrium plasmas with damped electrothermal waves.

The separation of the electrothermal mode and rate modes into different categories as implied by the labeling may be artificial. Their behavior is basically not very different. The major differentiation is that the effect of the magnetic field strength is more pronounced for electrothermal waves where a growth threshold appears.

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TABLE 1. Properties of argon plasmas considered in stability calculations

Case No.	p/p_{atm} [a]	$N_e (cm^{-3})$	Type [b]	$T_e (^oK)$	$\lvert H_{e(1)} \rvert$ [c]	M [d]	$L_c (m)$ [e]
1	.041	$7.5x10^{13}$	R	6000	7.76	1.55	.63
2	.41	$1.3x10^{16}$	R	8000	1.28	10.4	.22
3	.41	$5.0x10^{14}$	I	8000	3.96	3.5	.78
4	.41	$2.2x10^{14}$	R	6000	7.16	1.67	.41
5	.004	$2.2x10^{13}$	R	6000	8.72	1.38	.72
6	.004	$1.3x10^{15}$	R	8000	1.18	9.93	.23
7	.004	$5.0x10^{13}$	I	8000	6.64	2.15	.57
8	.004	$2.0x10^{12}$	I	6000	7.47	1.6	37.8
9	1.23	$4.0x10^{14}$	R	6000	5.8	2.02	.46
10	1.23	$1.0x10^{15}$	I	8000	3.53	3.85	.58
11	.004	$6.0x10^{14}$	I	10,000	4.17	3.78	.038
12	.041	$1.9x10^{15}$	I	10,000	4.12	3.92	.038

a $p_{atm} = 1.013 \times 10^5 \ N/m^2$

b I for ionizing, R for recombining

c Magnitude of the Hall parameter from Ohm's law

d Mach No.

e Interaction length

TABLE 2. Maximum growth factors, G, for argon plasmas from stability calculations

	Acoustic waves		Thermal-rate waves		
Case No.	G	$\zeta/2\pi$[a]	G	$\zeta/2\pi$[a]	Mode Type[b]
1	3.25	0.0	1.46	0.875	R
2	7.0	0.375	9.6	0.25	T
3	5.9	0.9375	14.0	0.375	R
4	2.85	0.0	3.25	0.875	R
5	2.9	0.0		Damped	
6	3.17×10^{2}	0.125		Damped	
7	4.3	0.0		Damped	
8	6.05	0.0		Damped	
9	2.72	0.0	3.85	0.875	R
10	4.88	0.875	1.9×10^{2}	0.375	R
11	5.25	0.0	1.005	0.75	T
12	4.9	0.0	1.2	0.25	T

a Calculations were made with 45° increments except for case 3 where 22.5° increments were used.

b R indicates rate, T thermal

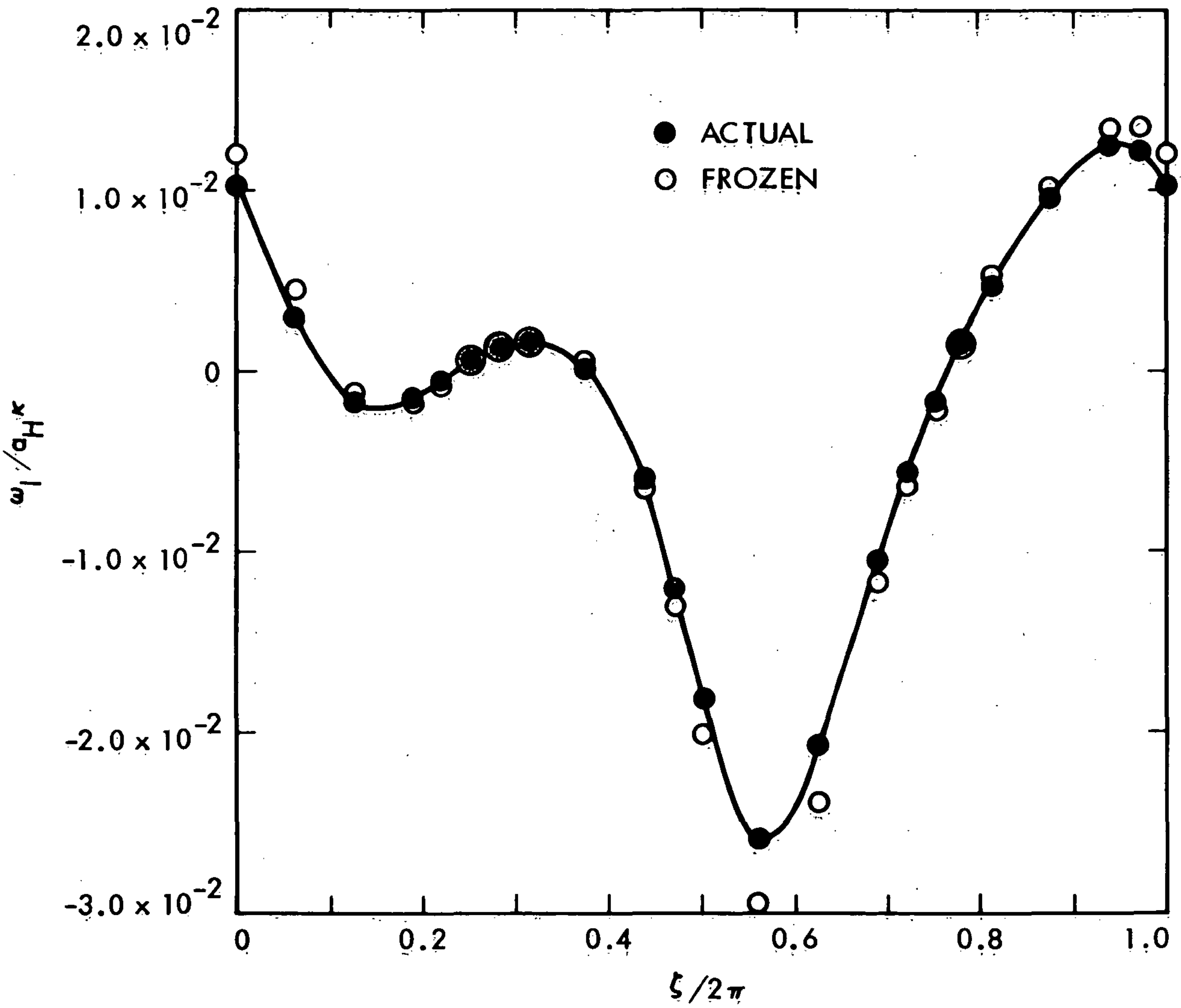

Fig. 1. Forward acoustic growth vs. angle ζ for case 3. The rate is given normalized by the acoustic frequency $a_H\kappa$. Label "actual" refers to numerical results

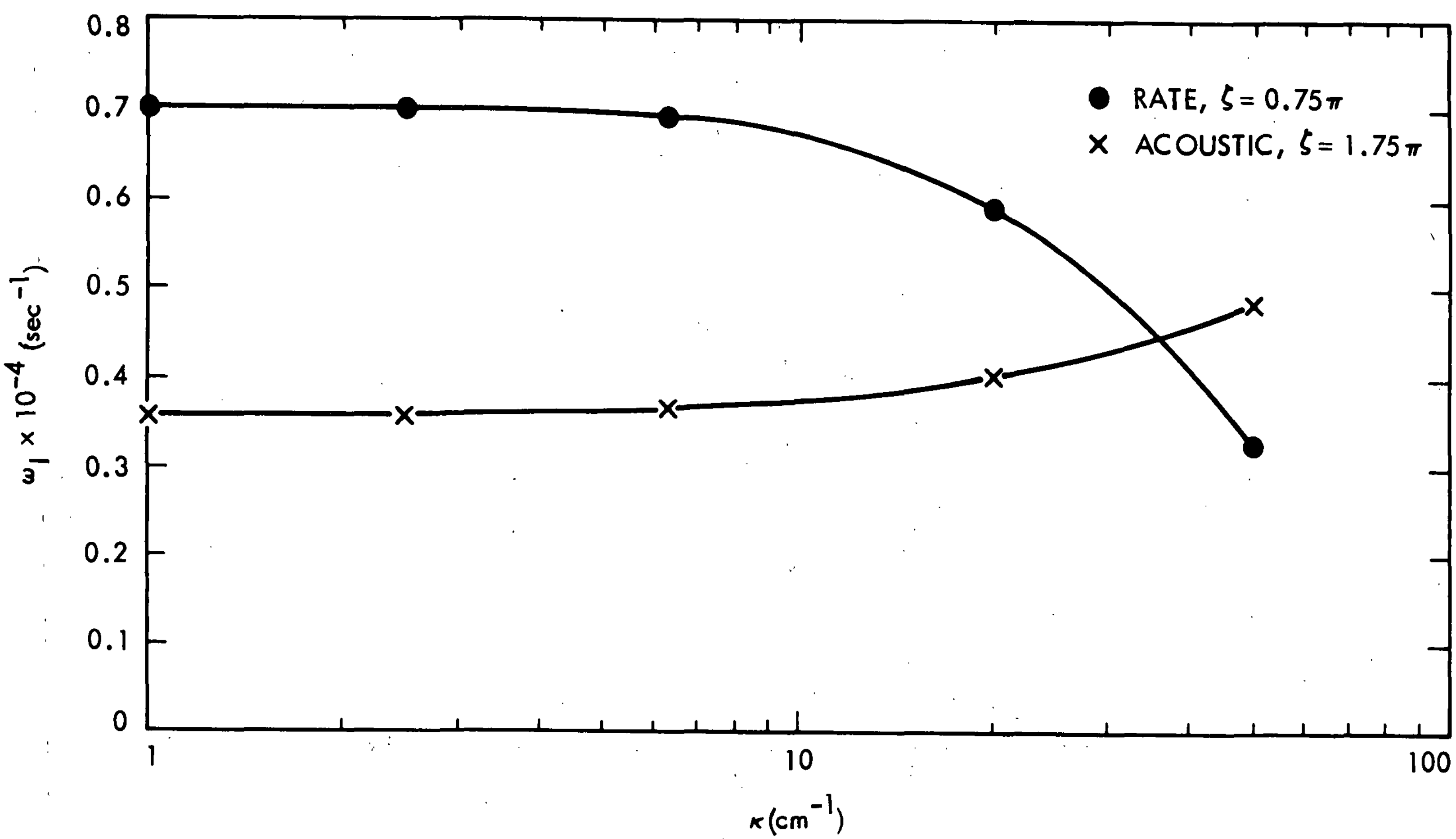

Fig. 2. Acoustic and rate growth vs. wave number for case 3, given δ

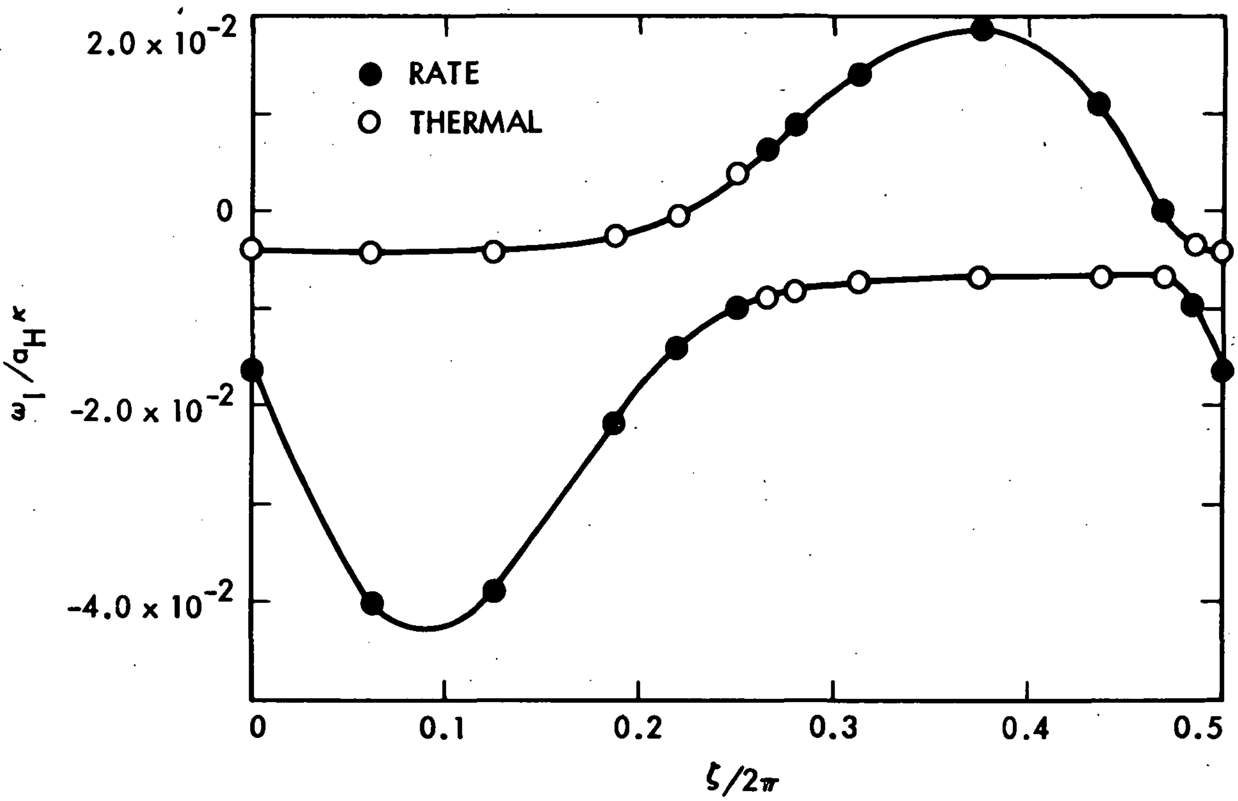

Fig. 3. Thermal-rate modes growth vs. angle ζ for case 3

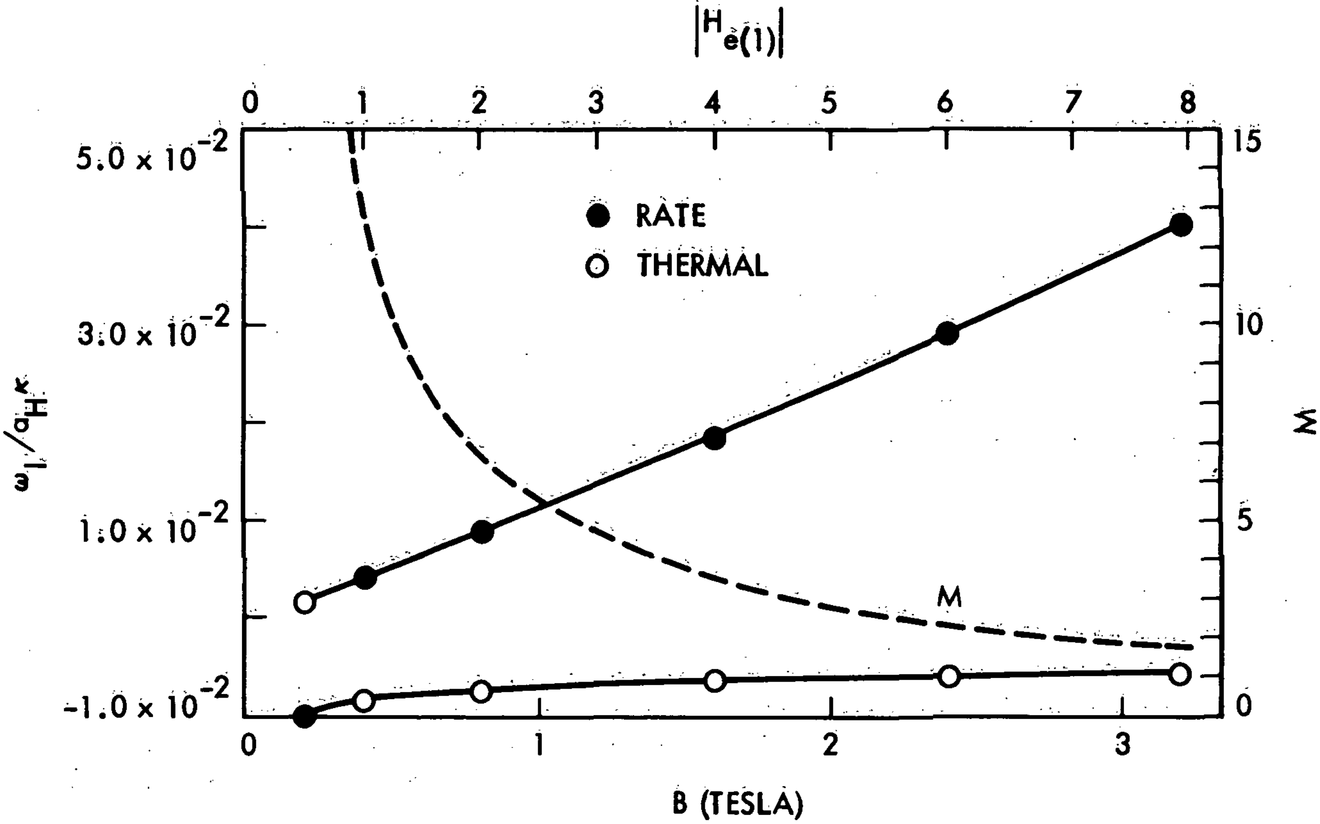

Fig. 4. Thermal-rate modes growth vs. magnetic field strength for case 3. The magnitude of the Ohm's law Hall parameter is shown on the top scale. The dotted line and right scale give the Mach number. Angle ζ = 0.74 π

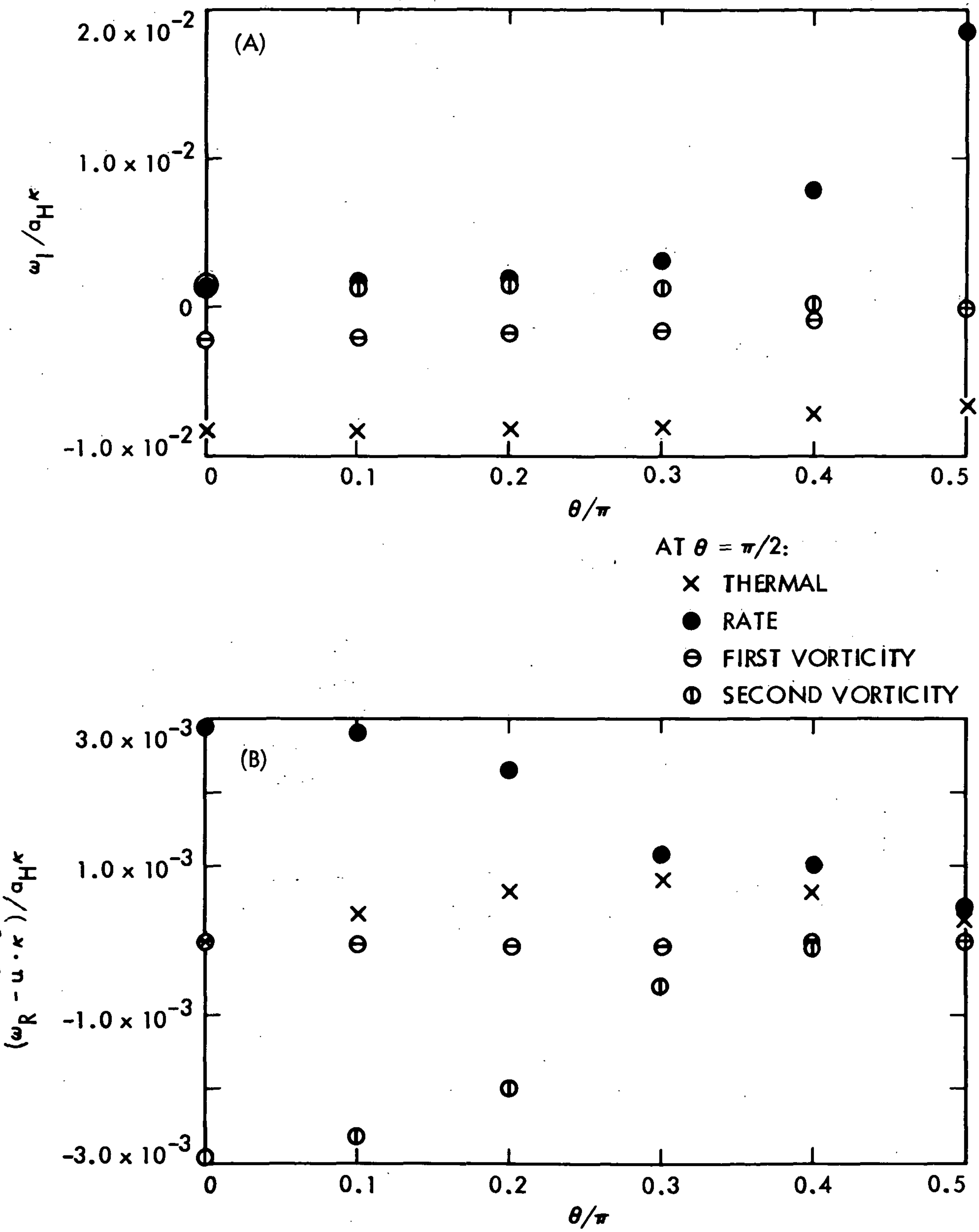

Fig. 5. Thermal-rate-vorticity modes eigenvalues vs. angle θ for case 3. Angle $\zeta = 0.74\ \pi$. Modes are labeled according to their behavior at $\theta = \pi/2$

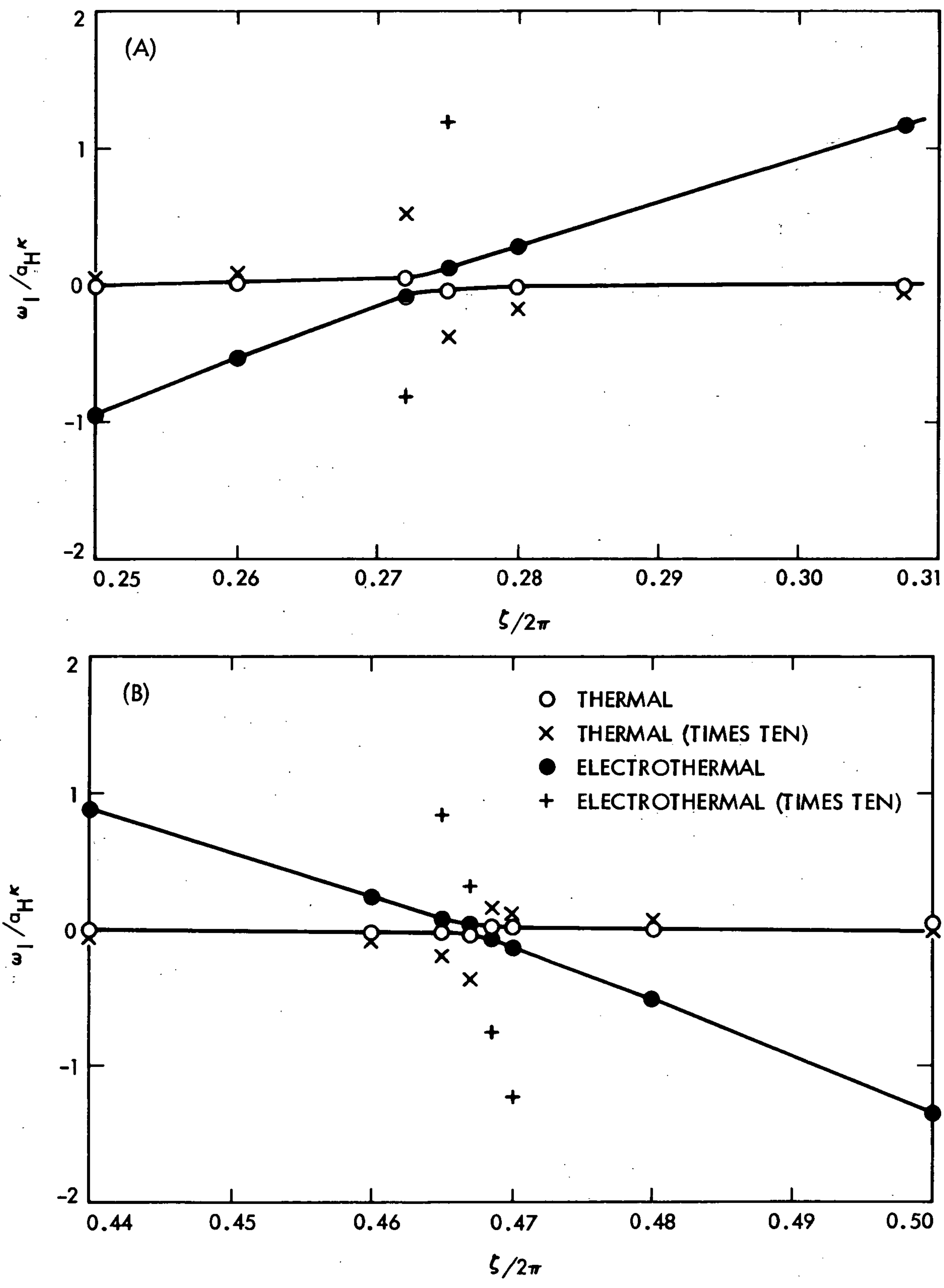

Fig. 6. Electrothermal and thermal growth vs. angle ζ for potassium seeded argon. Figure illustrates the mode type switching. $T_e = 4000°K$, $p = p_{atm}$, $\theta = \pi/2$

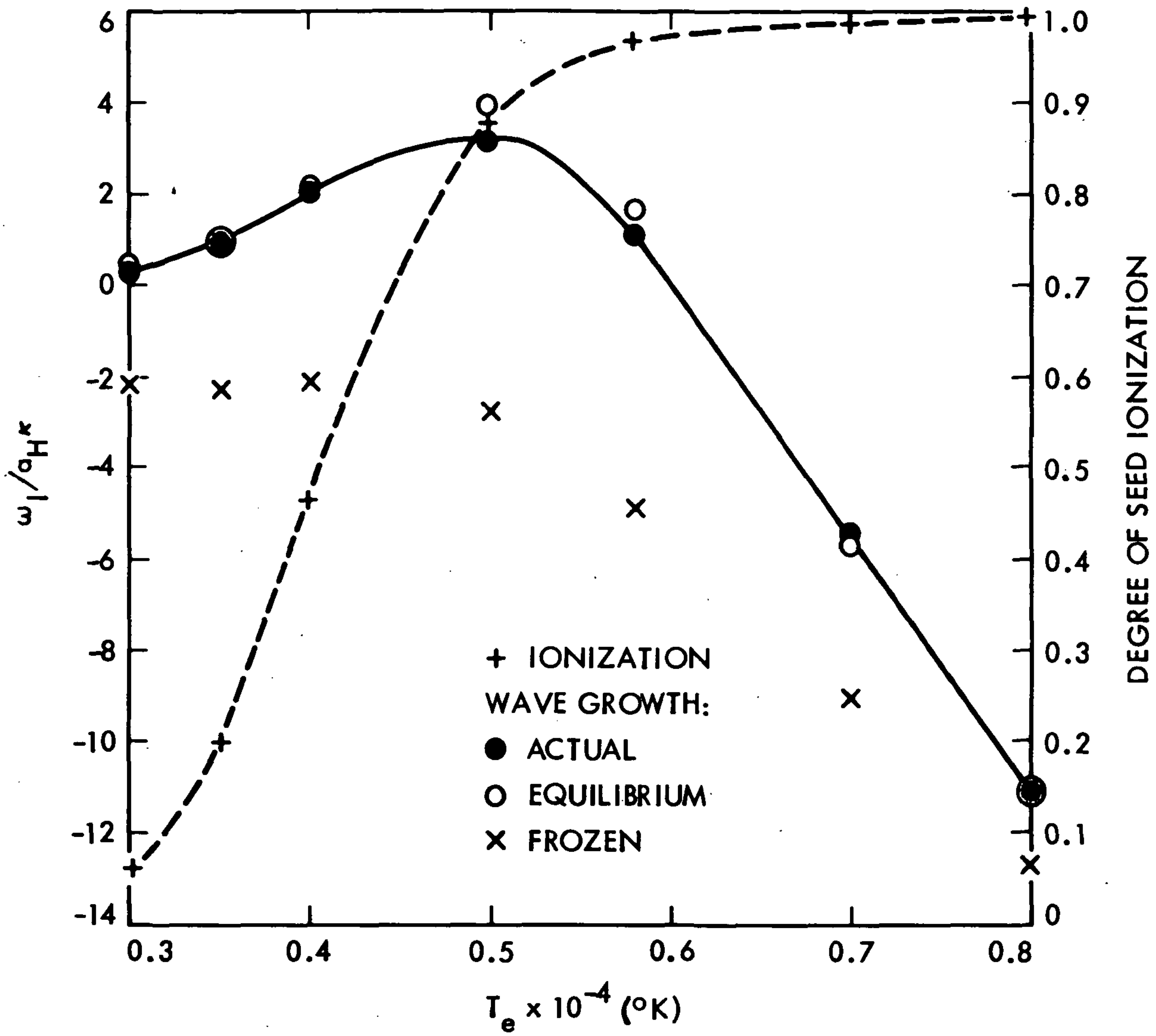

Fig. 7. Electrothermal growth and degree of seed ionization vs. electron temperature for potassium seeded argon.
$p = p_{atm}$ and $|H_{e(1)}| = 6.0$

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